

# Optimal health insurance contract : can moral hazard increase indemnity ?

David Alary  
SDFi, University Paris Dauphine

Franck Bien\*  
LEGOS, University Paris Dauphine

## Abstract

In this note, we generalize the results obtained by Barday and Lesur (2005) by considering a bivariate non separable utility function. We characterize optimal health insurance contracts. Moreover, we show that under moral hazard a sufficiently high risk aversion implies that the optimal coverage and the optimal preventive effort are higher than with perfect information.

**JEL Classification:** D82

**Key Words:** Moral hazard, Health insurance, bivariate utility function, health status, optimal contract.

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\*Corresponding author: LEGOS Université Paris Dauphine, Place de Lattre de Tassigny, 75775 Paris cedex 16 France  
Tel: [33](1) 44054427 / Fax: [33](1) 44054465 / E-mail: franck.bien@dauphine.fr

# 1 Introduction

In this paper, we characterize optimal health insurance contracts with moral hazard. In the context of insurance, moral hazard refers to the impact of insurance on incentives to reduce risk. An individual facing a risk can take actions to reduce this risk. Shavell (1979) has shown that preventive actions are discouraged by full coverage. The insurance contracts have to contain a deductible to reduce ex-ante moral hazard. The optimal contract will balance the risk-sharing benefit of greater insurance with the incentive benefits of less insurance. This is the main argument in favor of partial coverage for health insurance in most countries. However health risks have two major effects : a pecuniary one (health expenditures) and a health related one (health status). Moreover if the pecuniary risk is insurable, health status is usually uninsurable. Health status modifies the utility provided by wealth. Thus the way insurance contract deals with pecuniary loss is modified by health loss. Cook and Graham (1977) defined irreplaceable commodities as goods which modify the utility provided by wealth. Health appears to be a perfect example of such a good.

Thus, in order to deal with the problem of health insurance and moral hazard, we consider a bivariate non separable utility function as Rey (2003) who characterizes optimal insurance contracts with perfect information. Indeed, health status modifies expected utility level and modifies the marginal utility of wealth. Following Evans and Viscusi (1990) who show that marginal utility of income increases with health status in the case of severe injuries, we assume that the occurrence of a non-pecuniary uninsurable background loss reduces the marginal utility of wealth. Opposite to Rey (2003), we show the lack of the importance oh the sign of the variation of the marginal utility of wealth with respect to the health status for the insurers when they want that insureds have incentives to take preventive actions.

We show that the contract under asymmetric information provides a better coverage than the first best policy. This higher indemnity increases wealth in the illness state and decreases wealth in the healthy state due to the increasing premium. However policyholders value wealth more in the healthy state than in the illness state then they increase their effort in order to reduce the premium.

## 2 Framework

We consider a representative consumer. Their preferences are described by a bivariate utility function  $U(W, H)$  with  $W$  being the initial wealth and  $H$  being the health status.  $U$  is increasing and concave in each term  $U_1 > 0, U_{11} < 0, U_2 > 0, U_{22} < 0$ . The occurrence of a permanent loss in health can increase, decrease or can leave the individual's marginal utility of wealth unchanged. Three cases are possible :  $U_{12} < 0$  or  $U_{12} = 0$  or  $U_{12} > 0$ . In the field of health insurance, the marginal utility of income can increase or stay unchanged with health status in the case of severe injuries (Carthy et al., 1999 ; Sloan et al., 1998 ; Viscusi and Evans, 1990) and can decrease for minor injuries (Evans and Viscusi, 1991).

We assume that policyholders face a malignant risk. In case of illness, the consumer has to pay care expenditures  $L$  in order to recover his health status less a permanent loss  $\delta h > 0$ . This malignant risk appears with a probability  $p$ . The agent can decrease this probability taking preventive actions  $e \geq 0$  with a cost per unit of level effort. We assume  $p \equiv p(e)$  with  $p'(e) < 0$  and  $p''(e) \geq 0$ . The health risk is undeterministic since  $p(0) < 1$  and  $\lim_{e \rightarrow \infty} p(e) > 0$ . For the current problem, however  $p'(0) = -\infty$  and convexity of  $p(e)$  are sufficient conditions for the second-order conditions of the problem of the incentive compatibility constraint of insured. They choose a preventive effort level not only to reduce the financial risk but also to protect against the insurable health risk.

Insurers are competing in contracts to fulfill the demand for health insurance. A contract is a couple  $(\pi, q)$ , where  $\pi$  is the premium and  $q$  is the indemnity to be paid in case of illness.

## 3 Perfect information

With perfect information, the optimal contract maximizes expected utility subject to zero profit constraint, or participation from the insured :

$$\begin{aligned} \max_{q, \pi, x} & [(1 - p(x))U(w - \pi - cx, H) + p(x)U(w + q - \pi - L - cx, H - \delta h)] \\ \text{s.t.} & \pi = p(x)q \end{aligned} \quad (1)$$

The optimal contract propoerties are summarized in the following proposition.

**Proposition 1** *Perfect information contract has the following properties :*  
*if  $U_{12} < 0$  policyholders prefer over insurance*  
*if  $U_{12} = 0$  policyholders prefer full insurance*  
*if  $U_{12} > 0$  policyholders prefer underinsurance*

**Proof.** First order condition are

$$\begin{aligned} \frac{\delta \mathcal{L}}{\delta x} &= p'(x) [U(w + (1 - p(x))q - L - cx, h - \delta h) - U(w - p(x)q - cx, h)] \\ &\quad - [c + p'(x)q] \begin{bmatrix} (1 - p(x))U_1(w - p(x)q - cx, h) \\ + p(x)U_1(w + (1 - p(x))q - L - cx, h - \delta h) \end{bmatrix} = 0 \quad \text{FOC1} \\ \frac{\delta \mathcal{L}}{\delta q} &= (1 - p(x))p(x) \begin{bmatrix} U_1(w + (1 - p(x))q - L - cx, h - \delta h) \\ -U_1(w - p(x)q - cx, h) \end{bmatrix} = 0 \quad \text{FOC2} \end{aligned} \quad (2)$$

The second equation is equivalent to

$$U_1(w + (1 - p(x))q - L - cx, h - \delta h) = U_1(w - p(x)q - cx, h) \quad (3)$$

The coverage depends on the sign of  $U_{12}$ . To déterminate the value of  $q^*$  we evaluate FOC2 for full insurance. We obtain

$$\left. \frac{\delta \mathcal{L}}{\delta q} \right|_{q=L} = (1 - p(x))p(x) \begin{bmatrix} U_1(w - p(x)L - cx, h - \delta h) \\ -U_1(w - p(x)L - cx, h) \end{bmatrix}$$

The sign of this expression depends on the sign of  $U_{12}$ . Three cases are possible.

If  $U_{12} < 0$  then  $\left. \frac{\delta \mathcal{L}}{\delta q} \right|_{q=L} > 0$ . We deduce that  $q^* > L$ .

If  $U_{12} = 0$  then  $\left. \frac{\delta \mathcal{L}}{\delta q} \right|_{q=L} = 0$ . We deduce that  $q^* = L$ .

If  $U_{12} > 0$  then  $\left. \frac{\delta \mathcal{L}}{\delta q} \right|_{q=L} < 0$ . We deduce that  $q^* < L$ .

Rewriting *FOC1*, we obtain

$$\begin{aligned} & -p'(x) [U(w - p(x)q^* - cx, h) - U(w + (1 - p(x))q^* - L - cx, h - \delta h)] \\ & -p'(x)q^*U_1(w - p(x)q^* - cx, h) \\ & = cU_1(w - p(x)q^* - cx, h) \end{aligned}$$

Defining the optimal preventive effort. ■

When marginal utility of wealth is increasing in health status, policyholders want to reduce indemnity since a monetary unit provides a higher marginal utility in the healthy state rather than in the illness state due to the health status loss. However, the utility in healthy state is larger than the one in the illness state.

## 4 Asymmetric information

With ex ante moral hazard, the level of preventive effort cannot be contracted for since it is unobservable by the insurer. Rather than omitting the effort from the contract, we consider it as a contractual parameter and restrict the set of contracts with an incentive compatibility constraint. The optimal contract maximizes expected utility subject to the zero profit constraint of the insurer and the incentive compatibility constraint of the insured.

The optimal contract is solution of the following program:

$$\begin{aligned} \max_{q, \pi, x} \quad & [(1-p(x))U(w-\pi-cx, h) + p(x)U(w+q-\pi-L-cx, h-\delta h)] \\ \text{s.t.} \quad & \pi - p(x)q = 0 \\ & -p'(x) \begin{bmatrix} U(w-\pi-cx, h) \\ -U(w+q-\pi-L-cx, h-\delta h) \end{bmatrix} \\ & -c \begin{bmatrix} p(x)U_1(w+q-\pi-L-cx, h-\delta h) \\ +(1-p(x))U_1(w-\pi-cx, h) \end{bmatrix} = 0 \end{aligned} \quad (4)$$

We can substitute  $\pi$  using the zero profit constraint in the above program and so define  $x(q)$  as the solution in  $x$  to the incentive compatibility constraint. This yields expected utility as a function of  $q$  alone. Then, the optimal level of indemnity is solution of the following program

$$\max_q \begin{bmatrix} (1-p(x(q)))U(w-p(x(q))q-cx(q), h) \\ +p(x(q))U(w+q-p(x(q))q-L-cx(q), h-\delta h) \end{bmatrix} \quad (5)$$

The optimal coverage verifies the following condition

$$\begin{aligned} \frac{\delta EU(q)}{\delta q} &= -\frac{d\hat{x}}{dq}p'(\hat{x})q \begin{bmatrix} (1-p)U_1(w-p(x)q-c\hat{x}, h) \\ +pU_1(w+(1-p(x))q-L-c\hat{x}, h-\delta h) \end{bmatrix} \\ &+ (p(\hat{x})(1-p(\hat{x}))) \begin{bmatrix} U_1(w+(1-p(x))q-L-c\hat{x}, h-\delta h) \\ -U_1(w-p(x)q-c\hat{x}, h) \end{bmatrix} \\ &= 0 \end{aligned} \quad (6)$$

This first-order condition depends on the sign of  $\frac{d\hat{x}}{dq}$  and the difference between  $U_1(w-\pi-L+q-c\hat{x}, h-\delta h)$  and  $U_1(w-\pi-c\hat{x}, h)$ .

**Proposition 2** *The level of effort is increasing in the indemnity in the neighborhood of first best contract if*

$$-\frac{p'(\hat{x})}{cp(\hat{x})(1-p(x^*))} < -\frac{\begin{bmatrix} U_{11}(w+(1-\theta)q^*-L-c\hat{x}, h-\delta h) \\ -U_{11}(w-\theta q^*-c\hat{x}, h) \end{bmatrix}}{U_1(w-p(x)q^*-c\hat{x}, h)} \quad (7)$$

**Proof.** Fully Differentiating the incentive constraint and after computations, we obtain that

$$\begin{aligned} \frac{d\hat{x}}{dq} \Big|_{q^*} &= \frac{\begin{bmatrix} -p'(\hat{x})U_1(w-p(x)q^*-c\hat{x}, h) \\ +c \begin{bmatrix} p(\hat{x})(1-\theta)U_{11}(w+(1-\theta)q^*-L-c\hat{x}, h-\delta h) \\ -(1-p(\hat{x}))\theta U_{11}(w-\theta q^*-c\hat{x}, h) \end{bmatrix} \end{bmatrix}}{p''(\hat{x}) \begin{bmatrix} U(w+(1-\theta)q^*-L-c\hat{x}, h-\delta h) \\ -U(w-\theta q^*-L-c\hat{x}, h) \end{bmatrix}} \\ &+ cb2 \begin{bmatrix} p(\hat{x})U_{11}(w+(1-\theta)q^*-L-c\hat{x}, h-\delta h) \\ +(1-p(\hat{x}))U_{11}(w-\theta q^*-c\hat{x}, h) \end{bmatrix} \end{aligned} \quad (8)$$

The level of effort is increasing in the indemnity in the neighborhood of first best contract if and only if the above expression is negative.

Assuming the existence of preventive action with asymmetric information, we know that  $U(w+(1-\theta)q^*-L-c\hat{x}, h-\delta h) < U(w-\theta q^*-c\hat{x}, h)$  which implies that

$$\begin{aligned} p''(\hat{x}) \begin{bmatrix} U(w+(1-\theta)q^*-L-c\hat{x}, h-\delta h) \\ -U(w-\theta q^*-c\hat{x}, h) \end{bmatrix} \\ +cb2 \begin{bmatrix} p(\hat{x})U_{11}(w+(1-\theta)q^*-L-c\hat{x}, h-\delta h) \\ +(1-p(\hat{x}))U_{11}(w-\theta q^*-c\hat{x}, h) \end{bmatrix} < 0 \end{aligned} \quad (9)$$

Then  $\frac{d\hat{x}}{dq} \Big|_{q^*}$  is positive is and only if

$$\begin{aligned} -p'(\hat{x})U_1(w-p(x)q^*-c\hat{x}, h) \\ +c \begin{bmatrix} p(\hat{x})(1-\theta)U_{11}(w+(1-\theta)q^*-L-c\hat{x}, h-\delta h) \\ -(1-p(\hat{x}))\theta U_{11}(w-\theta q^*-c\hat{x}, h) \end{bmatrix} < 0 \end{aligned} \quad (10)$$

which is equivalent to

$$\begin{aligned}
& \frac{-p'(\hat{x})}{c} U_1(w - p(x)q^* - c\hat{x}, h) \\
< - & \left[ \begin{array}{l} p(\hat{x})(1 - \theta)U_{11}(w + (1 - \theta)q^* - L - c\hat{x}, h - \delta h) \\ - (1 - p(\hat{x}))\theta U_{11}(w - \theta q^* - c\hat{x}, h) \end{array} \right] \quad (11)
\end{aligned}$$

Considering the effect in the neighborhood of the first best contract implies that  $\theta = p(x^*)$  since  $\pi^* = p(x^*)q^*$ . Moreover we must have  $\hat{x} < x^*$  since  $\hat{x}$  is the solution of the incentive constraint where policyholders neglect the effect of their effort on the premium whereas  $x^*$  is the solution of the same constraint where policyholders take this effect into account. Then  $p(\hat{x}) > \theta$  and  $p(\hat{x})(1 - \theta) > (1 - p(\hat{x}))\theta$ .

This implies that

$$\begin{aligned}
& - \left[ \begin{array}{l} p(\hat{x})(1 - \theta)U_{11}(w + (1 - \theta)q^* - L - c\hat{x}, h - \delta h) \\ - (1 - p(\hat{x}))\theta U_{11}(w - \theta q^* - c\hat{x}, h) \end{array} \right] \quad (12) \\
> - & \left[ \begin{array}{l} p(\hat{x})(1 - \theta)U_{11}(w + (1 - \theta)q^* - L - c\hat{x}, h - \delta h) \\ - p(\hat{x})(1 - \theta)U_{11}(w - \theta q^* - c\hat{x}, h) \end{array} \right] \\
= -p(\hat{x})(1 - \theta) & \left[ \begin{array}{l} U_{11}(w + (1 - \theta)q^* - L - c\hat{x}, h - \delta h) \\ - U_{11}(w - \theta q^* - c\hat{x}, h) \end{array} \right]
\end{aligned}$$

Then a sufficient condition is that

$$< - \frac{\frac{-p'(\hat{x})}{cp(\hat{x})(1-p(x^*))}}{U_{11}(w-p(x)q^*-c\hat{x},h)} \quad (13)$$

■

**Corollary 3** *The optimal coverage is higher under asymmetric information than under perfect information.*

**Proof.** Evaluating equation 6 with the first best coverage, we obtain

$$\left. \frac{\delta EU(q)}{\partial q} \right|_{q^*} = - \frac{d\hat{x}}{dq} p'(\hat{x}) q^* \left[ \begin{array}{l} (1 - p(\hat{x}))U_1(w - \pi - c\hat{x}, h) \\ + p(\hat{x})U_1(w - \pi - L + q^* - c\hat{x}, h - \delta h) \end{array} \right] \quad (14)$$

If the above condition is verified then this expression is always strictly positive. It implies that the optimal coverage  $q$  verifying condition 6 is higher with asymmetric information than with perfect information. ■

Due to the asymmetry of information, insurers have to distort the coverage they offer from the first best coverage. However if the absolute risk aversion computed for healthy state is sufficiently higher than the absolute risk aversion in the illness state, this distortion should lead to an increase in the indemnity rather than a decrease as it should be the case in Shavell's model. This is due to the fact that an increase in the indemnity implies that the policyholder is richer in the unhealthy state than in the healthy state. However wealth is less valued in case of bad health than in case of good health. This gives incentives to increase prevention effort since it decreases its premium which increases wealth in the healthy state.

## 5 Conclusion

Health insurance should take into account health expenditures which is monetary losses i.e. also health status loss (disability for example). We introduce health status consideration in moral hazard problem on the insurance markets. As Bardey and Lesur (2005), we show that under conditions, the health status loss gives incentives to increase preventive action. Moreover, we show that under moral hazard, at the equilibrium, the coverage is higher than the first best one. This point is in contradiction with the well-known result of Shavell (1979) about the necessity of a deductible in insurance contracts under moral hazard and with result of Bardey and Lesur (2005) about the existence of first best contract under moral hazard..

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